Intermittent Search Processes: Chance Against Strategy

M. Moreau, O. Bénichou, C. Loverdo, P.H. Suet and R. Voituriez

Laboratoire de Physique Théorique de la Matière Condensée (UMR 7600)
Université Pierre et Marie Curie, 4 Place Jussieu, 75252 Paris France

Abstract. Intermittent search processes are widely observed in nature and in social activities. Simplified models show that they can be very efficient. Here we give new evidences of this efficiency. We discuss the conditions which can make intermittence favorable to the search, generally due to the imperfection of the detectors. We show that organized strategies, which are possible for searchers with sufficient abilities, in principle lead to shorter search times than random behaviors. Nevertheless, their advantage is not determinant, and random can be practically preferable because of its easier and more general use.

Keywords: intermittence, stochastic search, optimization
PACS: 87.23.-n

INTRODUCTION

Search processes are present at all scales in natural phenomena as well as in human activities. In many circumstances, a "searcher", which may be a molecule, a living organism or a social organization, has to find a given target by displacing its position, or its attention, till it discovers its objective. Search problems have been widely studied as optimization problems [1], often for practical motivations, like the search for submarines in 2nd world war, or in rescue operations. In general, information or signs concerning the location of the target play a predominant role for optimizing the search with respect to a given quantity, which can be, for instance, the energy needed for the search, or the number of targets reached in a given time.

The problem is significantly different in the case of hidden targets, when they can not be distinguished from a large distance and no indication allows one defining any non uniform target distribution probability. Then the searcher has to follow an arbitrary trajectory to prospect the territory until a target enters into the action field of the sensors which can detect it. However, careful scanning is generally a slow process, and it often occur that the searcher alternate phases of slow, detailed inspection with phases of fast displacements, which significantly reduce the perception abilities but rapidly lead to new territories. Although apparently not rational, such an intermittent behaviour is commonly observed for a large class of animals in quest of food [2, 3]. In the recent years, many articles have been published on these problems, mainly to find conditions which allow optimizing the search [4, 5, 6, 7, 8, 9, 10, 11]. In particular, it has been shown that intermittent processes, which alternate slow scanning and fast displacements, can allow for minimizing the first arrival time on the target [4, 5, 6, 7, 8, 9, 10, 12, 13].

Here we will extend these results and propose some general responses to questions which where formerly addressed by means of specific, restricted models: when can intermittent search be justified, instead of a systematic search? Is it possible that random be more efficient than a well determined strategy, and in some sense, that disorder be stronger than order? and eventually: how can the simplified models used for reasoning be useful in the huge complexity of actual situations?

To such general problems we shall not give complete answers, but just suggest some hints. First, we will consider the conditions which are necessary for justifying intermittent search processes and give examples where they are actually realized. Then, we will briefly review the results obtained on simple intermittent models, and mention various extensions which allow us considering more realistic situations. On this basis, and from reasonable extrapolations, we will eventually give some elements for answering the previous questions.

CP913, *Nonequilibrium Statistical Mechanics and Nonlinear Physics, XV Conference*
edited by O. Descalzi, O. A. Rosso, and H. A. Larrondo

SEARCH AND INTERMITTENCE

Hidden Targets, Finite Range Detectors And Random Search

We consider an immobile target located inside a finite region V of some space, assuming that no other information is available on its location, and that it can not be seen from a large distance a. As a consequence, the searcher has to move till the moment when its distance to the target is less than a, the detection range of activation. This motion should be decided arbitrarily, and in many cases it is chosen randomly.

The first search model with random trajectories was proposed by Vishwanathan and coworkers in 1999 and extended later [5]. In this model, based on experimental observations of albatrosses searching for food, the targets have a random distribution with uniform average density in infinite space. The searcher performs truncated Levy flights, consisting in a straight displacements in random directions. The distance l covered in one of these straight flights has a probability density $p(l) \propto l^{-\mu}$ for $l > a$ and $p(l) = 0$ for $l < a$, with $\mu > 1$. A target is found instantaneously if its distance from the searcher is less than the range a of the detectors. At the end of a flight, a new, independent Levy flight is immediately performed. The authors showed that, if the targets are left unchanged or immediately reconstituted after discovery, the average number of targets found during a given time (for constant velocity) is maximum for $\mu = 2$. This value corresponds to the maximum efficiency of the process in the present situation. However, if the targets are destroyed when they are discovered, the maximum efficiency is obtained for an infinite straight displacement, since this behaviour obviously allows to sweep the maximum volume during a given time.

In fact, targets often disappear after their discovery (except if they are distributed in patches), as it occurs for the preys destroyed by the predator or for victims of some accident when they are found by rescuers. Nevertheless, indefinite straight trajectories are not widely observed.

Imperfect Detectors

In fact, actual detectors need some time to analyze the data and do not allow for instantaneous detection, at least for hidden targets which can not be easily found. Thus their performances significantly decrease when the searcher moves rapidly, and detection can even need the searcher to stop completely. Thus efficient scanning must be slow, which may lead the searcher to interrupt it for moving rapidly to unexplored regions. Nevertheless, the relevance of this intermittent behaviour is unclear, and various models have been introduced to study if, and in what conditions, it can be justified [3, 6, 7, 8, 9, 10].

MODELS

We now consider simplified models alternating a slow motion allowing for detection, and fast displacements during which the target is not seen. The slow motion can be represented by a diffusion or a random walk: then the searcher moves relatively slowly, and he can detect a target within the range a of its detector. The fast displacements may be modeled by a ballistic motion, or even by a "teleportation" which randomly relocates the searcher in the search territory. Clearly, these are simplifying assumptions, but they represent typical cases and allow for analytical solutions and explicit answers to our questions. Generalizations will be considered later.

The duration of each phase, scanning and moving, is assumed to be an independent, stochastic variable, generally - but not always - following an exponential law. This is justified if the switching between the regimes is decided independently of searcher situation. In order to characterize the efficiency of the search, we use the mean search time, which is the first time a target is detected, averaged on the position of the target. The mean search time is primordial in most practical circumstances, and it does not depend whether the target is or is not destroyed by the searcher after discovery.

Diffusion And Ballistic Motion

The Example Of Foraging Predators: One Dimensional Model

Numerous observations of animals searching for preys show that they often use an intermittent strategy, or saltatory behaviour. During their fast phases, their velocity is approximately constant and their direction almost fixed: the turning angle between two ballistic phases is usually small. This example inspired the first, one dimensional model of intermittent search [4]. In this model, the searcher moves along an axis $0x$. It can be in two dynamical regimes: a *search regime*, labeled by 1 and modeled by a 1-dim diffusion with diffusion coefficient D, and a *move regime* 2, modeled by a ballistic motion with constant velocity v. The duration Ti of each regime i is an exponential stochastic variable with mean value τ_i. Preys are supposed to be regularly distributed along $0x$. They are found during regime 1 only, as soon as the searcher reaches them. The first arrival time of the searcher on a prey, starting in regime 1 from a given position, obeys the Chapmann-Kolmogorov backward equation [14] and can be computed classically. Its average on initial position yields the average search time S. In the low target density limit, it is found [4] that S scales as the inter-distance of the preys L, whereas it would scale as L^2 for a purely diffusive search. Thus it is clear that intermittence is favourable for the search.

Furthermore, it is shown that there is no global minimum of the mean search time S for finite values of τ_1 and τ_2: in order to minimize S both τ_1 and τ_2 should be as small as possible and satisfy the scaling law $\tau_2/\tau \propto (\tau_2/\tau)^{\nu}$, where $\tau = D/v^2$ is a characteristic time of the process, and the exponent ν depends on the conditions: if $\tau_1 \gg \tau$, $\nu = 2/3$: then $\tau_1 \gg \tau_2$ and the searcher spends more time searching than moving, which seems quite natural. But if $\tau_1 \ll \tau$, $\nu = 3/5$, and $\tau_1 \ll \tau_2$: then, the searcher *spends more time moving than searching* - although it can not find any target while moving! This non intuitive conclusion is not the only surprising one in these studies, but the results are in good agreement with experimental data on various, very different species including fishes, birds insects and lizards [2, 3, 4]. Thus in this example intermittence is an efficient strategy for minimizing the search time.

Memoryless Search In N Dimensions Space

The previous model is clearly limited by its simplifying assumptions. In particular, it should be extended to more general spaces, or in practice, to 2-dim and 3-dim spaces. On the other hand, the previous 1–dim searcher has space memory, since it remembers its velocity from one move phase to the other one. However, such memory can be impossible for a searcher with low abilities (molecules or elementary organisms). We now present a n-dim model for a searcher without such memory [9], which could be used, for instance, for addressing a diffusion limited chemical reaction where a species B reacts on immobile, catalytic sites A: $A + B \rightarrow A + C$.

The searcher now moves in a n-dim finite space V containing a target A. It alternates two regimes: during regime 1, it experiments a n-dim diffusion with coefficient D allowing for immediate detection when the searcher reaches the target; during regime 2, it follows a non-reactive ballistic motion with constant velocity v and arbitrary direction, randomly reoriented at each phase 2. The waiting times in both regimes are again exponential, independent variables with mean waiting times τ_i , $i = 1, 2$. Then the Chapman-Kolmogorov backward equations for the search times $S_1(x)$ and $S_2(x, v)$, starting from position x and velocity v in regimes 1 or 2, cannot be solved explicitly. However, it can be shown that they can be replaced by two diffusion-like equations which can be solved analytically in spherical geometry. For instance, in 2 dimensions, we consider that region V is a circle of radius b, and the target is a concentric circle of radius $a \ll b$. Then the explicit solution allows to compute the mean search time S and to treat the optimization problem analytically. It results that

- if $a < b \ll D/v$ intermittence is not favorable for minimizing the search time, and the best strategy is to stay in the diffusive regime indefinitely,
- if $a \ll D/v \ll b$ intermittence is (moderately) favorable. It allows minimizing the search time for finite values of τ_1 and τ_2 of the mean durations of the diffusion and ballistic regimes such that the diffusion and ballistic lengths are approximately equal. Then the minimum search time S_{min} is approximately half the search time in a purely diffusive regime: $S_{min}/S_{diff} \approx 0.5$.
- if $D/V \ll a \ll b$ there is also an optimal intermittent strategy with now $S_{min}/S_{diff} \ll 1$ if $b/a \gg 1$.

Diffusion And Teleportation

Protein Search For A Specific Site On DNA: One-Dim Model

Genetics events often depend on the interaction of a protein, or restriction enzyme, with DNA: the restriction enzyme cleaves the DNA chain at a specific sequence. This process it is of great importance, since it controls the kinetics of many genetics events. However, its mechanism remains mysterious. In fact, the cleaving rate is very fast, several orders of magnitude faster than if it was driven by diffusion. It has been suggested that this translocation process could alternate phases of diffusion along the chain, and jumps towards other sites of DNA.

Some years ago, we presented [4] a simple stochastic model for the localization of a protein on DNA based on these mechanisms. We assumed that the duration of each regime, 1-dim diffusion and 3-dim excursion, is an exponential, independent stochastic variable. Since the DNA molecule has generally a very intricate form, a small jump in space may correspond to a very large displacement along the DNA strand. Thus we can suppose that after an excursion in the bulk of any duration, the protein is randomly relocated on DNA: this is *teleportation*.

We showed that this model provides an optimal search strategy which can explain the observed fast rate of the process. In fact, the minimum search time is obtained when the mean durations of both phases are approximately equal, another non intuitive result of intermittence that was previously anticipated by simple considerations [15]. Furthermore, we proved [6] that the minimum search time S_{min} , compared to the search time S_{diff} in case of pure diffusion along the chain, is

$$\frac{S_{min}}{S_{diff}} = \frac{6}{L}\sqrt{D\tau} \tag{1}$$

(1). where L is the length of the DNA chain, τ is the average duration of each phase and D is the diffusion coefficient along the chain. Thus for a long DNA strand, the intermittent, 3-dim mediated strategy is extremely efficient and can accelerate the processes significantly. This conclusion and other results of the model agree with experimental observations.

Teleportation in n dimensions

It is possible to generalize the 1-dim model in a general, n-dim space. Then, teleportation, i.e. random relocation of the searcher anywhere in the (finite) accessible region V, independently of the jump duration, may be questioned. Nevertheless, it can be justified in various examples, in particular, if the average scale of the jumps is much larger than the average distance between targets. On the other hand, it is not necessary to assume that the duration of teleportation is an exponential variable, but only that it has a finite average τ_2. Supposing that the diffusive phase 1 has an exponential duration with average τ_1, it can be proved [16] that the mean search time is

$$S = \frac{1-\alpha-\langle\tilde{j}_1(1/\tau_1)\rangle_V}{\alpha+\langle\tilde{j}_1(1/\tau_1)\rangle_V}(\tau_1+\tau_2) \tag{2}$$

where α is the ratio $v(A)/v(B)$ of the volumes of A and B, and $\langle\tilde{j}_1(1/\tau_1)\rangle_V$ is the Laplace transform of the search time in regime 1 averaged on region V. If V and A are two concentric spheres with respective radius b and a, this quantity can be calculated exactly [16]. Then it is seen that in order to minimize the search time, τ_2 should be as small as possible, which is obvious; as for τ_1, two situations are possible, according to the value of τ_2 :

- if $\tau_2 > \tau_{2c}$ (τ_{2c} being a critical value that can be computed explicitly) S decreases continuously with τ_1 . Then the shortest search time is obtained for a purely diffusive search, and intermittence is not favourable.
- if, on the contrary, for $\tau_2 < \tau_{2c}$, the search time is minimum for a finite, non trivial value of τ_1: then intermittence is an efficient strategy which can make the search much shorter than pure diffusion.

These conclusions can be generalized to more general situations. It should be remarked that teleportation does not make use of any space memory of the searcher: this point is important for the following discussion.

ORDER, CHANCE AND STRATEGY

The previous models assume the searcher to have more or less developed abilities. We have seen that the technical performances of their detectors play an important role on the behavior to be adopted. On the other hand, in order to follow a determined trajectory the searcher needs to have space memory. It is the case in our first, 1-dim model: the searcher remembers its initial velocity when it begins a new ballistic phase. Similarly, it would need time memory if the durations of the dynamic regimes would obey non-exponential probability distributions, which will be considered later. We now discuss these memory effects.

Memory And Deterministic Exploration

Let us assume that the searcher is able to pre-determine certain trajectory Γ for exploring its territory, and to follow it between two successive scanning phases. Will the search be more efficient than if it randomly re-orients its trajectory after each search phase, as it was assumed in the n-dim ballistic model of previous section ?

In fact, an efficient trajectory Γ should obviously avoid self-crossing, since scanning the same region several times is time-consuming and useless for immobile targets and good detectors. If random is introduced in the "fast" trajectory because the searcher has no memory, the possibility of such self crossings cannot be avoided and the search time necessarily increases. This effect was indeed checked by numerical simulations in the 2-dim ballistic model [10].

On the other hand, multiple scanning of the same spot could have interest if we consider a new kind of imperfect detectors, i.e. detectors which can miss a target actually present inside their range. Then it might be useful to return at the same place. Nevertheless, it can be shown [17] that, if the detector has no time memory, the search time is always shorter for a self avoiding trajectory, and thus *a deterministic strategy* (without multiple scanning) during the fast regime *is always more efficient than a stochastic strategy*. This conclusion does not necessarily hold if the detector has time memory and if the probability to find the target increases at each passage.

Memory And Non Exponential Waiting Times

Searchers with time memory can chose non exponential waiting times for one or both regimes. Specific models should then be considered, since the waiting time laws should depend strongly on the example under study. In some cases, for instance teleportation (see previous section), the mean search time only depends on the mean duration of the regime. In general, however, this is not true. As a typical example, we considered the case of deterministic waiting times τ_1 and τ_2 when alternating diffusion and ballistic motion [9]. We have shown numerically that the search time is then slightly shorter than for exponential waiting times with the same mean durations.

This result seems intuitively justified: if the search is shorter for some definite values of τ_1 and τ_2 , it is reasonable to expect that efficient stochastic laws should have these average durations, and, furthermore, that the dispersion around these optimal values should increase the search time, or at least not decrease it.

Order And Disorder In Target Distribution.

A completely ordered target distribution is typically represented by a regular lattice, or in one dimension, by equally spaced targets. We used such a distribution in several works, or studied equivalently one target in a finite space with reflecting boundaries.

On the other hand, a typical disordered distribution is Poisson distribution, where each target is located independently of the other ones: different distributions should be motivated by specific examples. We studied Poisson distribution [18] in the case of alternating diffusion and ballistic motion. Comparing the results for distribution of equal density, both in numerical simulations and in some analytical approximations, we found [18] that the search time is slightly shorter for an ordered distribution.

Thus, once more the spatial memory implied by the regular distribution proves to be favorable to the efficiency of the search, without, however, producing a significant advantage for the searcher.

CONCLUSION

In this paper we have given new evidences, in particular for processes with teleportation, that intermittence can be a very efficient strategy in the search of hidden targets, thus confirming former results. In many circumstances, although not always, it allows decreasing the search time significantly, even when restricting the processes to simple diffusions and ballistic motions. Clearly, intermittence does not exclude using a larger class of processes, such as Levy flights, which have proved to be efficient in various cases: conjugating them with intermittence should presumably give excellent results in certain situations.

It has been shown that the interest of intermittence is in great part due to the imperfection of detectors and depends on the abilities of the searcher. As suggested by intuition, larger abilities can lead to better performances. In particular, space memory allows using a deterministic fast motion, which gives a shorter search time than stochastic fast motion. A similar advantage is obtained by imposing convenient deterministic waiting times to the alternating regimes, in case the searcher is able of time memory. On the other hand, an ordered distribution of the targets allows for a shorter search time than a disordered one - but this may be an inconvenient for the targets, which could, in this case, prefer disorder!

Thus we should conclude that the search is, in principle, more efficient using a relevant strategy than relying on random, which seems better for the sake of order and organization! We have given some quantitative arguments for this assertion which seems quite natural, but it should be pointed out that in all cases which could be treated accurately, the advantage of strategy is in no way determinant, and the search time is of the same order than in convenient random behaviors. In practice, the last remark leads to temper our conclusion: since finding the best strategy should depend on the specific problem to be treated, it is time and energy consuming. It depends on the abilities of the searcher, and it does not yield a definitive advantage. Thus it can be far simpler, and eventually preferable, to use a standard random behavior, if such standard behavior is available for the kind of search to be addressed. This can be the basis for further applied researches.

REFERENCES

1. J. R. Frost and L. D. Stone, *http.//www.rdc.uscg.gov/reports/2001/cgd1501dpexsum.pdf*
2. W.J. Bell, Searching behaviour, *the behavioural ecology of finding resources*, Chapman and Hall Animal Behaviour Series, 1991
3. W.J. O'Brien, H.I. Browman and B.I. Evans, *American Scientist* **78**, 152–157 (1990)
4. O. Benichou, M. Coppey , M. Moreau, P.-H. Suet and R. Voituriez , *Phys. Rev. Lett.* **94**, 198101–198105 (2005)
5. G.M. Viswanathan, S.V. Buldyrev , S. Havlin et al., *Nature* **401**, 911–914 (1999)
6. M. Coppey, O. Bénichou, R. Voituriez and M. Moreau, *Biophys. J.* **87**, 1640–1649 (2004)
7. O. Bénichou, M. Coppey, M. Moreau, P.-H. Suet and R. Voituriez, *J. Phys.: Condens. Matter* **17**, S4275–S4286 (2005)
8. O. Bénichou, M. Coppey, M. Moreau and R. Voituriez, *Europhys. Lett.* **75**, 349–354 (2006)
9. O. Bénichou, C. Loverdo, M. Moreau and R. Voituriez, *Phys. Rev. E.* **74**, 020102 (2006)
10. O. Bénichou , C. Loverdo, M. Moreau and R. Voituriez, *J. Phys.: Condens. Matter* **19**, 065141 (2007)
11. M. F. Shlesinger, *Nature* **443**, 281–282 (2006)
12. I. Eliazar, T. Koren and J. Klafter, *J. Phys.: Condens. Matter* **19**, 065140 (2007)
13. G. Oshanin, H.S. Wio, K. Lindenberg and S.F. Burlatsky *J. Phys.: Condens. Matter* **19**, 065142 (2007)
14. S.A. Redner, *A guide to first passage time processes*, Cambridge University Press 2001
15. M. Slutsky and L.A. Mirny, *biophys.J.* **87**, 4021–4035 (2004)
16. O. Bénichou, M. Moreau, P. H. Suet and R. Voituriez, Intermittent search and teleportation, *submitted*
17. M. Moreau, O. Bénichou, C.Loverdo and R. Voituriez, Random and strategy in stochastic search processes, *in preparation*
18. M. Moreau, 0. Bénichou, C. Loverdo and R. Voituriez, *EPL* **77** 20006 (2007)